$$e^{\underline{A}t} = \mathcal{L}^{-1}\left[(s\underline{I}-\underline{A})^{-1}\right]$$

$$e^{\underline{A}t} = P e^{Dt} P^{-1}$$

$$e^{\underline{D}t} = \begin{bmatrix} e^{d_1 t} & & & \emptyset \\ & e^{d_2 t} & & \\ & & \ddots & \\ \emptyset & & & e^{d_n t} \end{bmatrix}$$

$$e^{At} = S e^{Jt} S^{-1}$$

$$J = \begin{bmatrix} d_1 & 1 & 0 & & & & & \\ 0 & d_1 & 1 & & & & & \\ 0 & 0 & d_1 & & & & & \\ & & & d_4 & 1 & & & \\ & & & 0 & d_4 & & & \\ & & & & & d_6 & \ddots & \\ & & & & & & & d_n \end{bmatrix}$$

$$e^{Jt} = \begin{bmatrix} e^{d_1 t} & te^{d_1 t} & \frac{t^2}{2}e^{d_1 t} & & & \\ 0 & e^{d_1 t} & te^{d_1} & & & \\ 0 & 0 & e^{d_1 t} & & & \\ & & & e^{d_4 t} & te^{d_4 t} & \\ & & & 0 & e^{d_4 t} & \\ & & & & & \ddots \end{bmatrix}$$

EX:

$$A = \begin{bmatrix} 0 & 1 \\ 0 & -2 \end{bmatrix}$$

note: this can be done in matlab

expm(A)

matrix

but not w/ scalar "t"

$$e^{At} = ?$$

$$P = \begin{bmatrix} 1 & 1 \\ \lambda_1 & \lambda_2 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & -2 \end{bmatrix}$$

$$e^{At} = Pe^{\Lambda t}P^{-1}$$

$$= \begin{bmatrix} 1 & 1 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & e^{-2t} \end{bmatrix} \begin{bmatrix} 1 & 1/2 \\ 0 & -1/2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1/2(1-e^{-2t}) \\ 0 & e^{-2t} \end{bmatrix}$$

OR:

$$e^{At} = \mathcal{L}^{-1}\left[(sI-A)^{-1}\right]$$

$$sI - A = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ 0 & -2 \end{bmatrix} = \begin{bmatrix} s & -1 \\ 0 & s+2 \end{bmatrix}$$

$$(sI-A)^{-1} = \begin{bmatrix} \frac{1}{s} & \frac{1}{s(s+2)} \\ 0 & \frac{1}{s+2} \end{bmatrix}$$

$$\mathcal{L}^{-1}\left[(sI-A)^{-1}\right] = \begin{bmatrix} 1 & 1/2(1-e^{-2t}) \\ 0 & e^{-2t} \end{bmatrix}$$

CONTROL DESIGN

STATE FEEDBACK

$$\dot{x} = Ax + Bu \qquad x \in \mathbb{R}^n, \; u \in \mathbb{R}^m$$

the eigen values of A are the poles.

we want to change A

$$\dot{x} = \bar{A}x$$

A is changed by "Bu"

$$u = -Kx \quad \Big\} \text{ state feedback}$$

Hence the closed loop system becomes.

$$\dot{x} = Ax + B(-Kx)$$
$$\dot{x} = \underbrace{(A - BK)}_{\bar{A}}x$$

The state matrix of the closed loop system is $\bar{A} = (A - BK)$. The eigen values of the closed loop system are the eigen values of $\bar{A}$.

POLE PLACEMEN TECHNIQUE

Design K for the state feedback ($u = -Kx$) such that the eigen values of $(A - BK)$ are the desired eigen values.

note: MATLAB

$$K = place\left(A, B, \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_m \end{bmatrix}\right)$$

where $u_1 \ldots u_n$ are the desired eigen values.

$$K = acker\left(A, B, \begin{bmatrix} u_1 \\ \vdots \\ u_n \end{bmatrix}\right)$$

C.L. SYSTEM

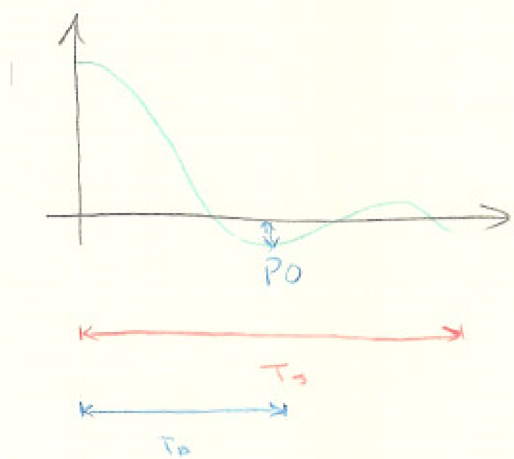
$$\boxed{\begin{aligned} \dot{x} &= Ax + Bu \\ x(t) &= e^{(A-BK)t}\, x(0) \end{aligned}}$$

here we can see if k is designed such that (A − BK) stable then.

$$\boxed{\lim_{t \to \infty} x(t) \to 0}$$

System Response is the same as controls I

EX:



EX: uncontrolable system.

$$\dot{x}_1 = x_2 + u$$
$$\boxed{\dot{x}_2 = -x_2} + \dot{x}_1$$

u does not have influence over $\dot{x}_2$

now $\dot{x}_2$ changes with u, it is no longer independent of the system.

$x(t) = e^{-t} x(0)$

how ever this system is self stabilising.

Under which conditions on A and B is it possible to design K, such that the eigen values of (A−BK) can be arbitarily assigned?

CONTROLIBILITY CONDITION

If the system is completely state controlable then we design K to arbitrarily assign the eigen values of (A−BK).

① $\dot{x} = Ax + Bu \qquad x \in \mathbb{R}^n$

System ① is controlable iff

$$\text{rank}(\mathcal{C}) = n$$

$$\mathcal{C} = [\,B \;\; AB \;\; A^2B \;\dots\; A^{n-1}B\,]$$

EX:

$$\mathcal{C} = \begin{bmatrix} x & x & x \\ x & x & x \\ x & x & x \end{bmatrix} \qquad \begin{matrix} \det \mathcal{C} \neq 0 & \text{rank} = 3 \\ \det \mathcal{C} = 0 & \text{rank} < 3 \end{matrix}$$

note: rank is the n value of the larges square sub matrix with det ≠ 0.

note: MATLAB $\quad \mathcal{C}$ is ctrb(A,B)

rank(ctrb(A,B))

Test of Rank

(1) If C_1 is square find $\det(C_1)$, if $\det(C_1) \neq 0$ then rank of C_1 is n.

(2) If C_1 is not square, compute $C_{1\,n\times nr}\,(C_1)^T_{nr\times n}$ or $C^T_{1\,n\times mn} \cdot C_{1\,mn\times n}$ and amply (1)